Hot-dog physics

(a) When a hot dog is cooked (boiled or fried), its skin often tears. When this happens, the tear is almost always axial (along the length of the hot dog) [Fig. 0.1, left]. Why do they essentially never tear along the circumference [Fig. 0.1, right]?

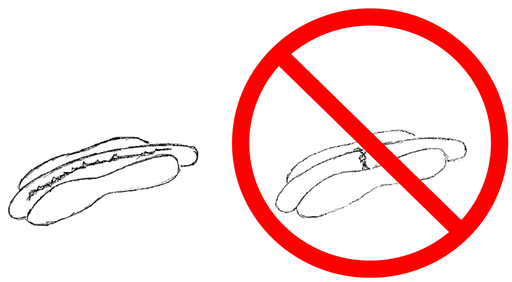

Fig. 0.1 Hot dogs often tear along the length (left), but essentially never along a circumference (right).

(b) A straight hot dog or sausage, when put on a hot grill, tends to curl into a "C" [Fig. 0.2]. Propose a theory for why this occurs.[1]

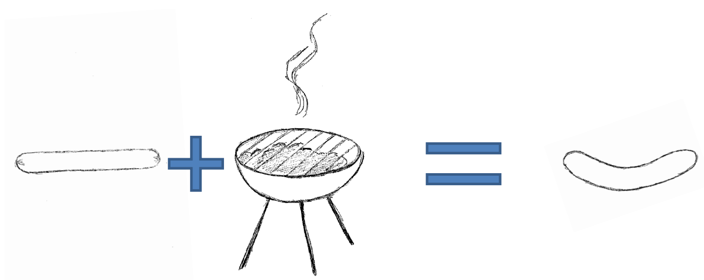

Fig. 0.2 When put on a grill, a straight hot dog curls into a "C."

[1] This question was suggested by Prof. John Close of the Australian National University.

Solution

(a) A hot dog is a long cylindrical shell (casing) made of small intestines of sheep or reconstituted collagen filled with some sort of meat or substitute filling. When the dog is cooked at high temperature, the pressure of the filling rises, which may lead to casing rupture.

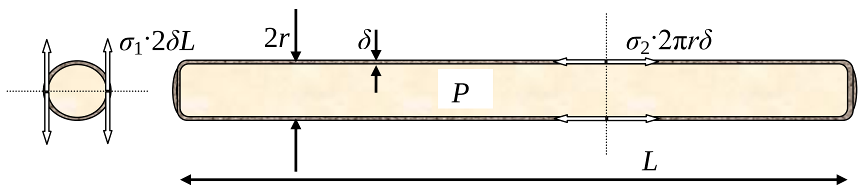

Fig. 0.3 Dimensions of a hot dog and total forces pulling the casing apart in two orthogonal directions, shown schematically with hollow arrows.

Let us assume that the length of the dog is L, the radius of the cylindrical casing is r, and the casing thickness is $\delta \ll r$ (Fig. 0.3). Assuming that the filling is at a uniform pressure P, the force that pulls two halves of the dog apart in a direction perpendicular to the hot-dog axis is $P \cdot 2r \cdot L$ for each half-dog. While the dog is intact, this force is balanced by the tension of the casing $2\sigma_1 \delta L$. Here σ_1 is the force orthogonal to the axis per unit area of the casing. Comparing the two expressions, we get $\sigma_1 = P \cdot r/\delta$.

Let us now look at the balance of forces along the axis. Here, we similarly have $P \cdot \pi r^2 = \sigma_2 \cdot 2\pi r \delta$, which yields for the casing tension in the axial direction: $\sigma_2 = P \cdot r/(2\delta)$, two times smaller than σ_1.

Presumably, the hot dog ruptures when tension reaches the strength limit of the casing material, which clearly happens at lower pressure for rupture parallel to the axis.

By the way, the tension difference also explains why some hot dogs increase their radius when cooked, while growing shorter: the casing expands according to the direction of highest tension, and the length has to shrink if the volume of the "meat" does not change that much.

(b) We need to confess here that the search for the solution to this problem led us to experimentation, often accompanied by subsequent consumption of the specimen [Fig. 0.4]. If you have never cooked a hot dog on a frying pan, this is actually a lot of fun. What happens is that the hot dog starts to spontaneously roll around the pan, as if it were alive (a rather dramatic site). Apparently, the reason for this is that bubbles form inside the casing near the hot surface, and there is rapid evaporation into the bubble, causing it to bulge. When this happens, the dog gets a push to roll (which happens in a random direction). This rolling effect confuses the results of our "curling"

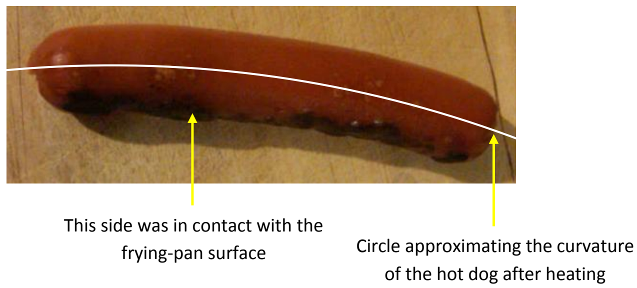

Fig. 0.4 A hot dog that was initially straight is seen here to be curled after cooking on a non-stick frying pan. During cooking, the dog was held lightly with a fork to prevent it from rolling (see text).

experiment. To mitigate this, we prevented rolling by slightly holding the dog in Fig. 0.4 while it was cooked on a frying pan (the figure shows the cooked dog removed from the pan and placed on the surface of a cutting board).

The result of the experiment shows that, apparently, curling results from the fact that the hotter side of the casing that is next to the hot surface shrinks compared to the opposite side of the casing, thus deforming the hot dog and causing curling. Measuring the radius of curvature of the dog using Fig. 0.4, we estimate the relative shrinkage of the casing close to the hot surface relative to the part of the casing on the opposite side of the hot dog as $r/R \sim 10\%$, where R is the radius of curvature of the cooked dog.